Ambiguity in the way of looking at geometrical figures

Ambigüedad en la manera de ver las figuras geométricas

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RESUMEN

Este trabajo analiza las diferentes maneras en que los estudiantes miran las figuras geométricas al resolver tareas geométricas y los diferentes tipos de razonamiento que tienen lugar en relación con los diferentes tipos de aprehensión figural, en el sentido de Duval, que se movilizan. El espacio de trabajo geométrico personal de los estudiantes de secundaria y bachillerato en Chipre se define con respecto a su forma de mirar las figuras y el tipo de razonamiento que producen.

PALABRAS CLAVE:

- *Aprehensión operativa*
- *Aprehensión perceptiva*
- *Razonamiento gráfico*
- *Razonamiento discursivo-gráfico*

ABSTRACT

This paper discusses the different ways the students look at geometrical figures in solving geometrical tasks and the different types of reasoning that occur in relation to the different types of figural apprehension, in the sense of Duval, that are mobilized. The personal Geometrical Working Space (GWS) of the students at lower and upper secondary school in Cyprus is defined in respect to their way of looking at figures and the type of reasoning they produce.

KEY WORDS:

- *Operative Apprehension*
- *Perceptual Apprehension*
- *Graphic Reasoning*
- *Discursive-graphic Reasoning*

RESUMO

Este artigo analisa as diferentes formas em que os alunos veem nas figuras geométricas na resolução de tarefas geométricas e diferentes tipos de raciocínio que ocorrem em relação aos diferentes tipos de apreensão figural, no sentido de Duval, que mobilizou. A geometria do espaço de trabalho pessoal de os estudantes da escola secundária inferior e superior em Chipre é definida com relação à maneira como você vê o número eo tipo de raciocínio que ocorrem.

PALAVRAS CHAVE:

- *Apreensão operatória*
- *Apreensão perceptual*
- *Inferência figural*
- *Raciocínio discursivo gráfico*

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RÉSUMÉ

Cet article examine les différentes façons dont les étudiants observent des figures géométriques pour résoudre des tâches géométriques et les différents types de raisonnement qui se produisent en relation aux différents types d'appréhension figural –dans le sens de Duval– qui sont mobilisés. L'espace de travail géométrique personnel des élèves du collège et du lycée en Chypre est défini par rapport à leur façon de voir les figures et le type de raisonnement qu'ils produisent.

MOTS CLÉS:

- *Appréhension opératoire*
- *Appréhension perceptive*
- *Inférence figurale*
- *Raisonnement discursivo-graphique*

1 Introduction

Despite the fact that nowadays much software exist for constructing geometrical figures, figures themselves are the blind spots for the teaching of geometry and for solving geometrical problems, because visualization is not truly succeeded by the students and the use of figures is often not very helpful for them to reach a solution (Duval, 1995). The issue of the students' acquisition of mobility in the vision of a figure – looking between areas, lines and points – led to working around the design and experimentation of situations where mobility is a key for the solution of the problem (Mathé, 2009). But how can students look at a change in the figures for getting access to geometrical concepts and problem solving?

In this paper, the ambiguity in the character of the geometrical figures due to different ways of looking at them is discussed by analyzing the students' reactions in geometrical tasks. Actually, the way the students look at geometrical figures is defined through a didactical analysis of their work in the tasks, which specifies the type of apprehension that was mobilized for the solution of the tasks. Furthermore, the results of this didactic analysis are used for setting the students' personal GWS (see section 4 of Introduction), based on the type of reasoning the students produced for solving the tasks. Thus, in this paper we discuss how the work of Duval (2005), regarding the use of figures in geometric thinking, permits the description of the organization of the components of the students' personal GWS. As we deal with the representation of geometrical figures, the emphasis is given on the real and local space of the GWS which is related to the cognitive procedure of visualization (see figure 1 of Introduction).

In addition, as the participants of the study are students from different educational levels, the results are discussed in relation to possible changes in their personal GWS after their transition from the lower to the upper secondary school. The description of the students' reactions according to the GWS can be used as an epistemological tool for teachers which can help them to identify their students' needs and properly adjust their teaching methods (Kuzniak & Rauscher, 2011).

2 Theoretical background

Geometrical figures are the representations possessing a central role in the geometrical activity. A figure merges three semiotic representations: magnitude, shape configurations and words naming the given properties. According to Duval (2005), the crucial issue in the learning of geometry is the separation between magnitude and visualization, because magnitude causes visual illusions and wrong perceptual estimation for the relations between figural units. Thus, the difficulties for most students are created due to a cognitive gap between two opposite ways of looking at figures and recognizing what they stand for: The natural perceptive way as for any visual representation of material objects or spatial organization (images, diagrams, plans, etc.) and the mathematical way for reasoning, defining, problem solving or proving (Duval, 2011). Perceptual recognition is sometimes misleading for the recognition of geometrical properties and, therefore, for the recognition of the geometrical objects represented. On the other hand, visualization is independent from magnitude and concerns only shape discrimination and configuration (Duval, 1995). Visualization is the simultaneous and immediate apprehension of a configuration as a whole. The heuristic use of figures is based on seeing and the interactions between seeing and reasoning, which are represented in a given figure.

More specifically, Duval (1995) distinguishes four apprehensions for a geometrical figure: perceptual, operative, discursive and sequential. In this contribution we focus on the first three types. Particularly, the perceptive way of visual recognition focuses exclusively on the most global shape or closed outline, according to the principles stated by the Gestalt theory, thus the recognition of other possible reconfigurations is excluded. The perceptive way is activated and reinforced when figures are used as objects that can be empirically observed and it can either help or inhibit the heuristic recognition (Duval, 2011). The operative apprehension is a form of visual processing that concerns geometrical figures and depends on the various ways of modifying a given figure. One way is the mereologic that refers to the division of the whole given figure into parts and the combination of them in another figure or sub-figures (reconfiguration). Within the operative apprehension the given figure becomes a starting point to explore other configurations that stem from the applications of these visual operations. The discursive apprehension deals with the valid use of properties for deducing. A figure is seen in relation to denomination or a hypothesis that make certain properties explicit. Perceptual apprehension cannot determine the mathematical properties represented in a drawing (Duval, 1995), so some mathematical properties must be given through speech (denomination and hypothesis). The absence of denomination and hypothesis in a drawing makes it an ambiguous representation and, thus, the properties that are seen are not the same for everyone (Duval, 1995).

As the figural register can visually demonstrate a property by itself, setting of significant moments for the development of mental images, the idea of graphical expansion (Richard, 2004) is not contradictory to that of the operative apprehension; it is complementary. But, unlike the complete statements of discourse, the figural register does not allow the production of comments or arguments. "The graphical expansion can be, under certain conditions, likened to a discursive-graphic reasoning" (Richard, 2004) and it can be expressed within the determined registers of semiotic representations. The discursive-graphic reasoning is a type of reasoning which articulates discursive and graphics proposals and actually is in line with the discursive reasoning as defined by Duval (1995), which refers to the coordination between registers. When a pupil moves from one utterance to a drawing, or from a drawing to a text, the coordination between the discursive and figural registers involves a cognitive activity of conversion which refers to the same object, even if the reference process to the ideal may be different. The figural inference is therefore the step of the discursive-graphic reasoning that changes the epistemic value, the semantics or the theoretical status of the discursive outcome (Richard, 2004).

❸ Methodology

The methodology of this research is based on a part of a preliminary analysis of a questionnaire administered in order to examine the students' geometrical figure apprehension in the sense of Duval's (1995) cognitive analysis. The tasks were administered to 616 students, aged 14 to 16, of lower (312 in Grade 9) and upper (304 in Grade 10) secondary schools in Cyprus. In this paper three tasks (see Appendix) are described and discussed. The main common point for the selection of these three tasks is the fact that they can be solved easily and rapidly by the involvement of the operative apprehension. In fact, the students can have an immediate access to the answer if they succeed in using the mereologic modification (the reconfiguration of the given figure) and, thus, base their answer on a graphic reasoning (Richard, 2004). However, these three tasks are significantly differentiated between them as well, because of the different didactic variables they involve. Thus, despite the fact that the solutions of these tasks were expected to be related to the operative apprehension of geometrical figures, associated to different types of cognitive procedures and figural apprehension, alternative solutions could also be provided by the students.

Consequently, besides the correctness of the students' final answer, it was very important to take into account the different cognitive procedures that were related to their solutions. An a priori analysis was made in order to trace the type of apprehension that was actually involved in the solution of the tasks. Therefore, the proposed analysis is qualified in terms of the didactic variables involved, the figural apprehension that was mobilized and the type of reasoning that occurred in each task.

3.1. *A priori analysis of the tasks*

The first task (OP1) is a classic one which was taken from a test constructed in France. The students had to compare the area of two figures. Actually, they were expected to perform a reconfiguration of Figure B (see Appendix, Figure 8) in order to form a figure similar to Figure A. This type of task is a classic task that takes into account a very common and very strong misconception (even for adults), which is actually that "if two figures are not identical, they do not have the same area". Based on the a priori analysis and the students' explanations, their correct answers (choice b) were categorized according to the type of apprehension that was mobilized. Three categories of answers were set:

1. *OP1me:* This category includes right answers that occurred from a graphic reasoning with the involvement of the operative apprehension, in which the reconfiguration was explicit. The term "*explicit*" denotes a certainty that students have made a reconfiguration either by drawing extra lines in the figures or by the verbal description of the modifications made mentally (Figure 6).
2. *OP1pe:* In this category the students' right answers occurring from vision (counting the squares) were grouped. These answers occurred mainly by the perceptual apprehension, whose mobilization was expected to be facilitated by the presence of the grid. The use of the grid is a didactical variable intuitively introducing the notion of magnitude (measures) and enhancing the confusion between area and perimeter.
3. *OP1da:* In this case the students' right answers that occurred from a different visual approach were grouped. There were answers based on the outline of the figures (e.g "*figure 1 is a rectangle, but figure 2 is not*", "*the lines forming figure 1 are straight, but in figure 2 they are not*") or by using measures and calculations.

The second task (OP2) was constructed by one of the authors and was used in previous relevant researches. The students had to find the length of one side of the rectangle, based on the fact that its area is equal to the area of the trapezium. Students were expected to find the answer only by graphic reasoning and make a reconfiguration of the trapezium (a rectangle is formed by moving one of the two triangles and joining them properly). In this task the didactic variable of the notion of magnitude was introduced directly, with the presence of numbers on the given figure, in order to examine also whether the students would be influenced from the usual didactic contract (Brousseau, 1990) and proceed to an algorithmic solution using a formula. The students' answers were discriminated into two categories:

1. *OP2me:* this group includes the right answers that occurred from the operative apprehension and the reconfiguration of the figure was explicit, either by a verbal description or by drawing for showing the modifications on the given figure.
2. *OP2pe:* in this case the right answers that occurred from the perceptual apprehension are grouped. The students focused on the recognition of the global shape and/or its sub-figures and used a formula for finding the area of the trapezium. Then, they used the formula of the area of the rectangle for finding the missing side of the figure. Such answers occurred from the influence of the didactic variables (magnitude, didactic contract) and included the combination of the figural and the discursive register (discursive-graphic type of reasoning).

The third task (OP3), which is taken from Euclid and has been also used and discussed from Duval, included a rectangle divided to different sub-figures (triangles and rectangles) and the students had to compare the area of the two shadowed sub-figures. This task does not include the notion of magnitude, thus for its solution the reconfiguration of the given figure is necessary. This task is, however, clearly related to the deductive argumentation, or in other words, to the discursive-graphic reasoning, as the students have to use also their theoretical knowledge. The students' correct answers were categorized as below:

1. *OP3me:* in this category, the right answers that occurred from a mereologic argument were grouped. In fact, in the figure ABCD, the diagonal AC divides the rectangle into two equal right-angled triangles (ADC and ABC). Each of these triangles include two other right-angled triangles which occur from the division of two rectangles included in the figure ABCD respectively. Thus, from the triangles ADC and ABC equal parts are subtracted. Consequently rectangle 1 and rectangle 2 have an equal area. For such a solution procedure the operative apprehension must be mobilized for discriminating the different reconfigurations and

realizing the relations between these subfigures. The students were expected to go through these reconfigurations in order to solve the task, but the discursive register was also involved, so the discursive-graphic reasoning occurred.

2. *OP3pe:* in this case the students' right answers were justified with compensatory relations between the two shadowed rectangles (e.g. "*rectangle 1 is long and narrow, but rectangle 2 is short and wide*", "*if rectangle 1 is divided into two equal parts and these two parts are joined, then we get rectangle 2*"), due to the mobilization of the perceptual apprehension.
3. *OP3da:* this group includes right answers resulting from mere vision and thus a different approach including measuring the sides of the two shadowed rectangles for calculating their area.

4 Results

4.1. *Descriptive analysis*

The students' right answers in the tasks in relation to the approach they used is presented in Table I. We shall note that the percentages of answers in each category are not very big, because of the limited proportion of students that provided an explanation for their answer. This is an important matter to take into account; however it exceeds the purpose of this paper. In the first task the prevailing type of reasoning was the graphic, as the mobilization of the operative apprehension gives the most correct answers. The number of students that succeed though the mobilization of the perceptual apprehension is not very big, thus the influence of the grid does not appear to be so intense. In the second task the greater percentage of correct answers occurred from the mobilization of the perceptual apprehension and the use of formulas. In this case the influence of magnitude and the didactic contract is more evident, which brought about the combination of the figural and the discursive register. Thus, the discursive-graphic reasoning was found to be more effective than reconfiguration for succeeding a right answer. In the third task the number of right answers occurring from the predominance of mere vision is greater than the success through visualization. The involvement of perception or the students' tension for measurements gave more right answers than the reconfiguration of the given figure in combination to the theoretical knowledge. Thus, in this task mere vision prevailed the students' discursive-graphic reasoning.

Table I
Percentages of students' answers in relation to the type of apprehension involved

	TASK 1		TASK 2		TASK 3	
	Grade 9 %	Grade 10 %	Grade 9 %	Grade 10 %	Grade 9 %	Grade 10 %
Right Answer	62.82	71.38	32.05	35.86	53.21	66.78
Operative apprehension (reconfiguration)	22.12	23.68	6.09	8.55	6.73	9.21
Perception	12.18	18.09	22.44	25.99	9.62	13.49
Different Approach	14.42	14.47	–	–	12.82	10.53

4.2. *Hierarchical clustering of variables*

To learn more about the students' personal GWS our data were analyzed using the hierarchical clustering of variables with the computer software C.H.I.C. (Bodin, Coutourier, & Gras, 2000). This method of analysis determines the hierarchical similarity connections of the variables. From the similarity diagram of the 9th graders' responses (Figure 1) three distinct similarity clusters can be identified, indicating three different groups of students, according to the type of figural apprehension they mainly mobilize for the solution of the tasks. Cluster 1 comprises of answers that occurred through the application of the mereologic modification. Cluster 2 includes mainly answers related to the perceptual apprehension, whereas the third cluster is formed by answers linked to the use of a different approach, mainly including measurements and calculations. The clusters are formed in a similar way also in the similarity diagram for the 10th graders (Figure 2). However, in this grade the students do not display the same coherence regarding the mobilization of the operative and the perceptual apprehension, as there are similarity relations between the two types of solutions in clusters 1 and 2.

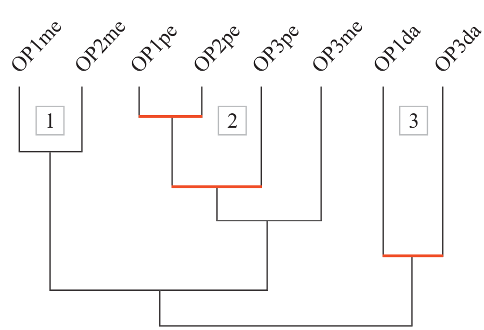

Figure 1. Similarity diagram of grade 9 students' approaches in the tasks

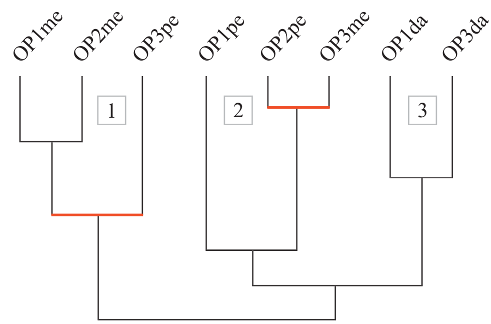

Figure 2. Similarity diagram of grade 10 students' approaches in the tasks

4.3. *An interpretation in terms of personal GWS*

The results of the similarity diagrams can be further interpreted in terms of the GWS, based on the type of reasoning included in the students' answers. Table II presents the type of reasoning that corresponds to each kind of answers.

TABLE II
The re-categorization of the students' answers in relation to the type of reasoning involved

Variable	Type or reasoning	New variable
Task 1		
OP1me	Graphic Reasoning	GR
OP1pe	Vision – Perception of Objects	VP
OP1da	Vision – Calculations	VC
Task 2		
OP2me	Graphic Reasoning	GR
OP2pe	Discursive - Graphic Reasoning	DGR
Task 3		
OP3me	Discursive – Graphic Reasoning	DGR
OP3pe	Vision – Perception of Objects	VP
OP3da	Vision – Measurements	VM

Based on this re-categorization according to the type of reasoning, the hierarchical clustering of variables was repeated (Figures 3 and 4), for expressing the previous results in relation to the students' personal GWS. What is extracted from the new diagrams is that, even when examining the results through the type of reasoning, the three groups of students are still distinguished. In the first group the students that fall are the ones who reason only in a graphic way, the second group includes the students who base their answers on the perception of objects and who also do discursive-graphic reasoning, whereas the last group consists of students who look at figures through mere vision and thus use measurements and calculations.

What is also observed is that the graphic reasoning is compartmentalized from the discursive-graphic reasoning, but the visual approaches including measurements and calculations are also compartmentalized from the rest types or reasoning. Therefore, the students do not succeed to coordinate these types of reasoning for the solution of geometrical tasks and each type seems to function independently. In addition, the visual approach related to the perception of objects is mainly linked to the discursive-graphic reasoning, especially in grade 9. This relation indicates that the students that approach the figure perceptually need to involve also the discursive register for supporting their answer. On the other hand, for the students that succeed visualization through the heuristic exploration of the figure, the system of reference does not seem to be necessary, as their solution is based on the reconfiguration of the given figure. Therefore their answers could be considered as a “proof without words” (Richard, 2003).

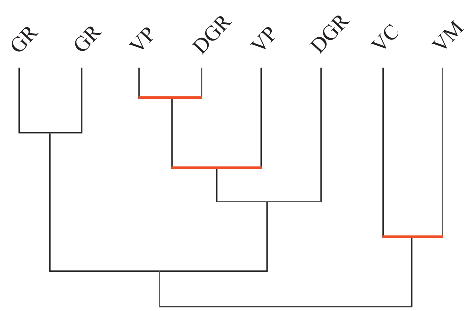

Figure 3. Similarity diagram of grade 9 students’ approaches in the tasks

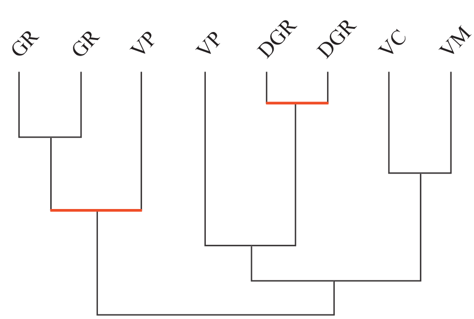

Figure 4. Similarity diagram of grade 10 students’ approaches in the tasks

Based on these results, an effort to describe the students' personal GWS is made in Figure 5. In fact, our results indicate that within the epistemological plane, in the component of real and local space, three main approaches appear. For the solution of geometrical tasks the students can either mobilize the operative apprehension, the perceptual apprehension or perform measurements/calculations. The mobilization of the operative apprehension is related to the graphic reasoning and, thus, during this cognitive process, the video-figural genesis (see Introduction, Figure 4) proceeds according to the process of visualization and representation, respectively turned to the real and local space (Coutat & Richard, 2011). But this visualization process must be distinguished from mere vision or perception of objects, as it nourishes the intuition of the properties and it sometimes helps to establish cognitively the validity of these properties (see Introduction, section 2.2). On the other hand, the mobilization of the perceptual apprehension appears to lead the students to a discursive-graphic reasoning, thus, during their solution procedure, the discursive-graphic genesis is made, which brings the students to the production of a relevant proof. The involvement of vision, which leads to the use of measurements and calculations, is not related to a particular type of genesis. Consequently, different types of genesis are found to happen for the different groups of students within their personal GWS, according to the type of figural apprehension that is mobilized for their solution and correspondingly to the type of reasoning that is produced.

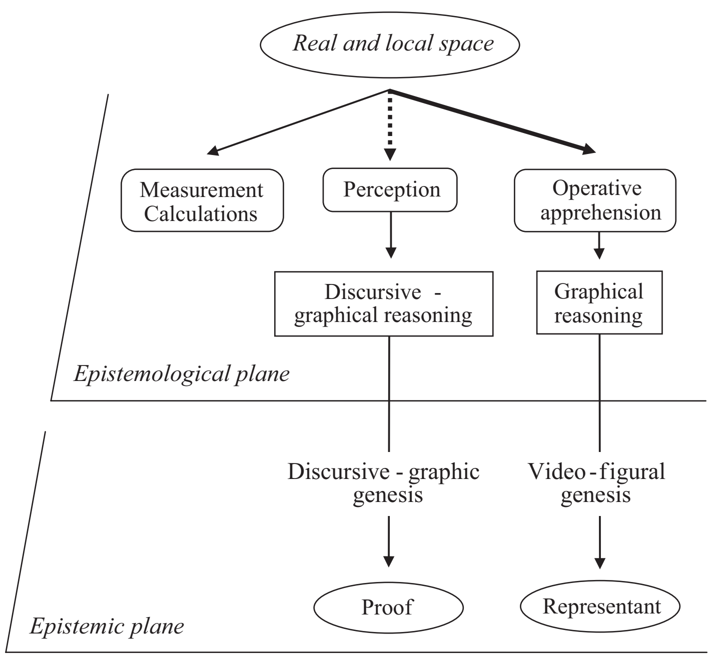

Figure 5. The students' personal GWS

❺ Conclusions and discussion

In this contribution an effort was made based on empirical data to show that what a geometrical figure shows to a students' eye is different according to the way the student succeeds to look at the figure. The ambiguous character of geometrical figures influences the way the students approach them and use them for solving geometrical tasks. Based on the way the students are influenced by the ambiguity of geometrical figures, some conclusions are drawn about their personal GWS, focusing on the component of real and local space.

From the analysis of the students' answers, it was obvious that the same given figure in a task could be seen in different ways due to the mobilization of a different type of figural apprehension. The involvement of these different figural apprehensions was, in some cases, enhanced from the different didactical variables included in the tasks. Therefore, different kinds of reasoning were identified and different types of genesis were found to happen within their personal GWS. Actually, when the operative apprehension is mobilized, the given figure is the source for the students' solution, because some relevant modifications bring the students to graphic reasoning, without involving any theoretical knowledge. Alternatively, the given figure can be the basis for the students' answers, as the students recognize perceptually some relevant properties in the figure and then use a proper theorem or formula, coming to the discursive-graphic reasoning. This case is in line with Duval (1995), who supports that, in any geometrical situation, "the perceptual recognition of properties must remain under the control of statements, as what the perceived figure can represent is determined by speech acts and depends on the deductive dependence between statements". On the other hand, the mere vision of the given figure can lead the students into using other approaches as well, such as measuring and calculating.

Thereafter, the ambiguity in the way geometrical figures can be seen is also revealed from the diversity in the students' way of reasoning, as for the same given figure the graphic reasoning, the discursive-graphic reasoning or more visual ways of thinking can occur. But do teachers understand that the ambiguous status of figures can cause difficulties to their students and how can they help the students realize that there are different ways to see the same figure? So, we should reflect on a new approach for introducing geometry in primary and secondary levels, whose principle would be that the awareness of the different ways of looking at figures is prior to the knowledge of the classical basic figures (Duval, 2011). To this end, teaching geometry should be focused in making students able to see flexibly in geometry, as it is possible to initiate the development of skills which enhance the mobility of seeing (Mathé, 2009). Thus, tasks about discriminating various figural units must be separated from the ones about magnitudes.

As the graphic reasoning is a step towards the discursive-graphic reasoning, the students' graphic reasoning must be developed by enhancing the operative apprehension of figures, which is complementary to such type of reasoning (Richard, 2004). In the absence of reference, momentarily, the processes resulting from the video-figural genesis play a heuristic role of first order (Coutat & Richard, 2011) and the students' ability in the heuristic exploration of figures helps them identify the relevant theorem for solving a task. Therefore, the flexible manipulation of figures within the figural register can bring the students more easily in combining effectively the discursive register also and use their reference knowledge, based on the realization of the relations between the different parts of the figures. The success of the exploration of the figure in the context of a given problem will depend of the relationship between the operative apprehension of the figure and a discursive set of inferences which mobilizes a network of definitions and theorems (Duval, 1995).

Regarding the comparison between the two groups of students, we observe no striking difference between the two educational levels. No actual improvement is observed between the lower and the upper secondary school students regarding the graphic or the discursive-graphic reasoning. Therefore, the teaching of geometry in Cyprus does not seem to help the students develop the different types of reasoning and genesis within the vertical planes of their personal GWS after their transition to the next educational level. Consequently, the enhancement of the operative apprehension must be continued until the upper secondary school and at this level specific tasks about the discursive apprehension must also be proposed, so that the students may use properties and theorems in a mathematical way.

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Appendix

Task 1

Underline the right sentence and explain your answer.

a) Fig. A has bigger area than Fig. B
b) Fig. A has equal area with Fig. B
c) Fig. A has smaller area than Fig. B

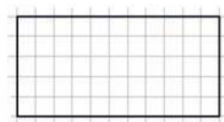


Fig. A


Fig. B

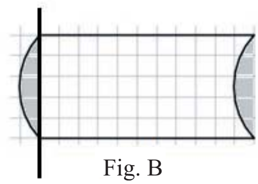
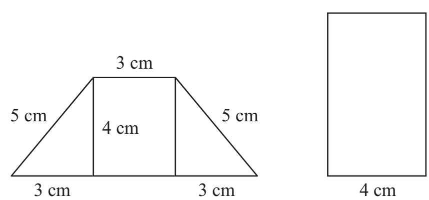


Fig. B

Figure 6. The explicit reconfiguration of figure B in task 1

Task 2

The trapezium and the rectangle have equal areas. Find the length of the missing side of the rectangle and explain your answer.



Task 3

The figure ABCD is a rectangle. Look at the shadowed rectangles 1 and 2 and choose the correct answer. Then justify your choice.

a. Rectangle 1 has bigger area than rectangle 2.
b. Rectangle 1 has equal area with rectangle 2.
c. Rectangle 1 has smaller area than rectangle 2.