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MATHEMATICAL MODELING FROM THE EYES OF PRESERVICE TEACHERS

RESUMEN

Utilizando las opiniones de los docentes en servicio como base, este estudio busca arrojar luz sobre el proceso seguido por los docentes en servicio en la enseñanza del modelado matemático a estudiantes de secundaria. El grupo de estudio estaba compuesto por 18 maestros de pre-servicio de matemáticas de la escuela intermedia, cada uno de los cuales fue seleccionado mediante muestreo intencional. Durante el período de investigación, los participantes viajaron en grupos a las escuelas donde debían realizar su práctica. Las lecciones fueron grabadas en video, y los participantes compartieron estas grabaciones y sus experiencias en el aula con sus compañeros. Como resultado del análisis, los hallazgos del estudio se agruparon en cuatro temas principales: (i) opiniones con respecto a actividades, (ii) opiniones con respecto a maestros en servicio, (iii) opiniones con respecto a estudiantes y (iv) opiniones con respecto a maestros de matemáticas.

PALABRAS CLAVE:

- *Modelación matemática*
- *Profesores de matemática en servicio*
- *Actividades de modelado matemático*
- *Ejecución de actividades*
- *Educación matemática*

ABSTRACT

Using preservice teachers' (PTs) opinions as its base, this study seeks to shed light on the process followed by PTs in teaching mathematical modeling to middle school students. The study group was composed of 18 middle school mathematics PTs, each of whom was selected using purposeful sampling. During the research period, PTs travelled in groups to the schools where they were to perform their practicum. Lessons were video recorded, and PTs shared these recordings and their classroom experiences with their peers. As a result of the analysis, the study's findings were grouped into four main themes: (i) opinions regarding activities, (ii) opinions regarding preservice teachers, (iii) opinions regarding students, and (iv) opinions regarding mathematics teachers. Discussion of these findings revolved around both teacher training and mathematical modeling, which then led to several recommendations being made.

KEY WORDS:

- *Mathematical modeling*
- *Preservice mathematics teachers*
- *Mathematical modeling activities*
- *Execution of activities*
- *Mathematics education*

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RESUMO

La discusión de estos hallazgos giraba en torno a la formación del profesorado y el modelado matemático, lo que llevó a varias recomendaciones. Usando as opiniões dos professores de preservice como base, este estudo procura esclarecer o processo seguido pelos professores de preservice no ensino de modelagem matemática para alunos do ensino médio. O grupo de estudo foi composto por 18 professores de matemática do ensino médio, cada um dos quais foi selecionado por meio de amostragem intencional. Durante o período da pesquisa, os participantes viajaram em grupos para as escolas onde deveriam realizar seu estágio. As aulas foram gravadas em vídeo e os participantes compartilharam essas gravações e suas experiências em sala de aula com seus colegas. Como resultado da análise, as conclusões do estudo foram agrupadas em quatro temas principais: (i) opiniões sobre atividades, (ii) opiniões sobre professores de preservação, (iii) opiniões sobre estudantes e (iv) opiniões sobre professores de matemática. A discussão dessas descobertas girou em torno da formação de professores e da modelagem matemática, o que levou a várias recomendações.

PALAVRAS CHAVE:

- *Modelagem matemática*
- *Professores de matemática preservice*
- *Atividades de modelagem matemática*
- *Execução de atividades*
- *Educação matemática*

RÉSUMÉ

En se fondant sur les opinions des enseignants de pré-requis quant à sa base, cette étude vise à mettre en lumière le processus suivi par les enseignants de pré-requis dans l'enseignement de la modélisation mathématique aux élèves du secondaire. Le groupe d'étude était composé de 18 enseignants du secondaire en mathématiques du secondaire, dont chacun a été sélectionné à l'aide d'un échantillonnage ciblé. Pendant la période de recherche, les participants se sont rendus en groupe dans les écoles où ils devaient effectuer leur stage. Les leçons ont été enregistrées sur vidéo et les participants ont partagé ces enregistrements et leurs expériences en classe avec leurs pairs. À la suite de l'analyse, les résultats de l'étude ont été regroupés en quatre thèmes principaux: (i) les opinions concernant les activités, (ii) les opinions concernant les enseignants préposés à l'entretien, (iii) les opinions concernant les élèves et (iv) les opinions concernant les professeurs de mathématiques. La discussion de ces résultats a tourné autour de la formation des enseignants et de la modélisation mathématique, ce qui a conduit à plusieurs recommandations.

MOTS CLÉS:

- *Modélisation mathématique*
- *Professeurs de mathématiques de base*
- *Activités de modélisation mathématique*
- *Exécution d'activités*
- *Enseignement des mathématiques*

1. Introduction

Recent studies have emphasized the need for students to be exposed to mathematical modeling (MM) at the earliest stages of school and have shown that by dealing with real-life problems, students are able to make significant developments to their mathematical and social skills (Asempapa, 2015; Doerr & English, 2003; English & Watters, 2004). As a result, just as MM is included in the curricula or educational standards of many developed countries, MM skills are considered basic skills that students are expected to master (Common Core State Standards Initiative [CCSSI], 2010; National Council of Teachers of Mathematics [NCTM], 2000). This importance notwithstanding, MM is neither sufficiently integrated into teaching environments, nor is it properly taught to students. Since teachers are the most important element in overcoming this dilemma, they must first gain sufficient knowledge of MM and then create rich learning environments that nurture students' active participation in modeling activities (Borromeo & Blum, 2009).

1.1. *MM in Mathematics Education*

While models are defined as a system incorporating rules, relationships, operations, and other components to explain and construct a complicated and difficult-to-understand system (Lesh & Doerr, 2003), modeling refers to the process followed to create a model (Sriraman, 2005). MM is the process of using math to construct a model that produces answers to individuals' important real-life questions and then of testing and implementing this same model (see: Niss, Blum, & Galbraith, 2007). MM is one of the fundamental components of mathematics education in the world-wide accepted *Program for International Student Assessment* [*PISA] 2012 Assessment and Analytical Framework*. As such, the fundamental objectives and goals that the current mathematics education paradigm seeks to impart to students need to be evaluated from a MM perspective.

In mathematics education, four separate teaching objectives are considered important for MM: (i) behavioral objectives, (ii) process objectives, (iii) affective objectives, and (iv) cognitive objectives (Lesh & Doerr, 2003). Behavioral objectives include knowledge, input-output, and provisory courses of action. For example, skills included in educational programs expected to be acquired by students are considered behavioral goals. From a MM perspective, however, since knowledge is likened to a living organism instead of to a machine, acquiring behavioral objectives is not a primary objective in and of itself. Process objectives focus on problem-solving strategies used in lessons, with these strategies being taught independent of content. In MM, however, problem-solving strategies reflect how students interpret situations requiring solutions. That said, affective objectives represent students' beliefs about their lessons as well as their attitude, motivation, emotions, and values (Lesh, Zawojewski, & Carmona, 2003). Finally, cognitive objectives include models and conceptual systems used to explain, construct, and test mathematically complicated systems. From a MM standpoint, emphasizing other goals without including cognitive goals renders mathematics education into nothing more than rote memorizing rules that are then used to find the correct answer without actually making any calculations. Just as these four teaching objectives must be a component of students' experiences and mathematical interpretations, models must also be understood as approaches developed to aid students in making these interpretations (see: Lesh & Doerr, 2003).

On the other hand, the effective teaching of MM in a classroom setting is generally performed through the execution of relevant activities. Just as modeling activities aid students in using important mathematical concepts, they also offer teachers the opportunity to discover what their students think about mathematics and to shed light on their mathematical development (English, 2003; Mousolides, 2009). Accordingly, it is essential that MM activities gain a foothold in educational environments.

1.2. *Mathematical Modeling Activities*

Mathematical modeling is an activity in which real-life problems are solved using mathematical tools. (Vos, Hernandez-Martinez, & Frejd, 2020). However, not every activity in a real-life context can be a mathematical modeling activity. In the modeling activity, not all the data necessary to solve the problem are given in the problem. It is essential for the student to make some assumptions in order to solve the problem as in real life (Borromeo, 2018). In these activities, calculation is not at the forefront, and all necessary thinking and planning skills are at the forefront before the calculations begin. In addition, Modeling activities not only help students use important mathematical ideas, but also help teachers uncover their students' thoughts and provide an opportunity to develop an understanding of their mathematical development. (English, 2003; Mousolides, 2009). The theoretical and philosophical structure underlying this thought is based on American pragmatists like Dewey, who defended obtaining information by transforming concrete experiences into abstract concepts through observation, thought, and new inferences, as well as on modern philosophers like Piaget and Vygotsky (see Lesh & Doerr, 2003; Lesh & Sriraman, 2005a, 2005b). This is because this philosophical approach argues that the forms of thinking that are required always occur by using and integrating much more than a single discipline, single course book, or single subject area.

In the last 10-15 years, many empirical studies have been conducted to investigate the teaching and learning processes of mathematical modeling. Much of this work has focused on teaching students at the K-12 level. The most well-known of these is the work of Lesh and Doerr (2003) (Borromeo, 2018). These studies argue that mathematical modeling education should begin at a very early age for students to understand how mathematics is needed in their real lives. Below, mathematical modeling applications are given from the perspective of students from the eyes of Lesh and Doerr (2003).

1.3. *Mathematical modeling for students*

Modeling activities require students to develop mathematical models that allow them the opportunity to work in groups to make sense of their mathematical experiences and to focus on their structural characteristics, such as relationships between models, operations, and components (Chamberlin, 2004). Considering this importance, modeling activities can be executed in a planned manner using basic mathematical concepts previously taught; yet, this requires students to first develop a mathematical model for themselves (Lesh, Carmona, & Post, 2002). Lesh and Doerr (2003) express the principles that need to be taken into consideration while conducting modeling activities as follows: (i) The concepts and mathematical notions aimed to be taught need to be discussed with students beforehand, (ii) If necessary, a warm-up activity or activities seeking to increase the real-world application and meaningfulness of the problem's context for students should be completed, and (iii) Activities pertaining to the usability of models developed by students should be done following the main activity's completion (Lesh et al., 2000). Modeling activities following these principles are conducted with students in groups of three to five individuals. During group work, each student first offers his or her own interpretation of the problem, and these interpretations are then discussed in groups. During discussions, students are given ample opportunity to improve their communication skills. During these group discussions, students attempt to compromise on the most appropriate model. What is sought is a mathematical model able to be used to solve a problem identified in a given scenario or by an individual in need of an answer. So that their peers may benefit from the most suitable model possible, students must clearly describe their thought processes and explain their solutions (Chamberlin & Moon, 2005). This way, group work is more than simply a social activity for students; it develops into a cognitive environment (Burkhardt, 2006).

The stages that students go through during modeling activities are summarized by Yoon (2006) in Table I.

TABLE I
Stages of Modeling Activities

Warm-up	*Problem Solving*	*Presentation*
Students read and discuss relevant introductory articles on the problem context.	Students work in groups to develop solutions to the given problems.	Students share their solutions mostly through writing but sometimes visually.

Adapted from Yoon, C. (2006)

Generally speaking, MM activities' in-class execution and student-teacher roles greatly resemble the environments recommended in the constructivist approach (Burkhardt, 2006). The modeling process is realized in student-centered environments not only in which students are given the opportunity to learn from their mistakes in a constructive manner and are guided to consider different ideas and methods to solve problems but also in which teachers manage the entire process, including assessments, in an appropriate manner (Kunter & Voss, 2013). It is also known that these learning environments improve students' modeling skills (Blum & Borromeo, 2009).

1.4. *Mathematical modeling in terms of teachers and teacher candidates*

There are few studies in the field of mathematical modeling for teachers and prospective teachers (Borromeo, 2018; Yang, Schwarz & Leung, 2021). These studies report that teachers misunderstand the nature of modeling before receiving a training in mathematical modeling, and they also have difficulties in modeling processes (Manouchehri, 2017; Paolucci & Wessels, 2017; Yang, Schwarz & Leung, 2021). Blum & Borromeo (2009) reported that teachers are an indispensable element for mathematical modelling. Accordingly, teachers should provide ample opportunities for students to connect, encourage students' independence, and adopt an effective and student-centered classroom management. Niss, Blum & Galbraith (2007) report that the content of education designed for teachers should include modeling experiences that match what they are expected to teach. However, there are also studies suggesting an educational design that combines theory and practice. Accordingly, first the participants are given a theoretical training and they are provided to experience the current modeling activities. Then the participants design their own activities and apply it to the target audience. Finally, all applications are evaluated. (Borromeo, 2013; Borromeo, 2018; Borromeo & Blum, 2013; Lesh & Doerr, 2003).

1.5. *MM in the Turkish Mathematical Curriculum*

Together with the introduction of a new mathematics education program, a constructivist approach in which student-centered activities where students produce solutions to real-life problems was adopted in 2005 in Turkey, which served to revamp the entire education paradigm and its practices (Ersoy, 2006). Over the subsequent years, mathematics curricula were readjusted to reflect this change. In the Middle School Curriculum published in 2013 (Ministry of National Education in Turkey [MONE], 2013a), for example, MM skills were included among the basic skills expected to be acquired by students. During the same period, an elective class called *Mathematics Applications* incorporating MM activities was added to the middle school curriculum (MONE, 2013b). In the more recently updated 2018 Mathematics Curriculum, acquiring mathematical literacy skills, of which MM holds a central position, were included among the basic objectives of mathematics education (MONE, 2018). Generally speaking, a strong emphasis on mathematics' being an integral part of real life and a worthwhile endeavor pervades throughout the newly restructured mathematics curricula (Şen, 2017).

2. Research Objective and Importance

Studies have concluded that MM activities are not performed at the desired level in classroom settings (Blum & Borromeo, 2009; Burkhardt, 2006). Although performing MM activities is included as a basic objective of mathematics curricula, one of the most important reasons for their exclusion from being used in classroom settings is that it is difficult for both teachers and students to perform related activities. (Blum & Borromeo, 2009; Frejd & Ärlebäck 2011). In any case, every step of the above-mentioned modeling process constitutes a an aptly-named cognitive hurdle that must be overcome by students (Galbraith & Stillman, 2006). Upon comparison, the percentages corresponding to the number of correct answers to questions on the international PISA requiring MM skills were found to be lower than those of other questions on the same test (Turner, Dossey, Blum, & Niss, 2013). Additionally, teachers also need to have acquired a variety of skills, like being able to properly guide students in completing complex modeling processes, have mathematical and non-mathematical knowledge, and be able to conduct activities that facilitate the acquisition of new ideas (Blum, 2015).

Accordingly, teachers and students must know just what is required of them play an important role in activities so that activities may be effectively realized and gain real-life applications. In addition to this, it is important that preservice teachers (PTs) have worked with modeling activities and have gained valuable experience using them so that they may understand the difficulties involved in completing them and how they should direct their future students in their interactions with unfamiliar processes that incorporate open-ended questions and for which they must take into consideration numerous variables to solve. As such, the present study has examined three aspects of activities actively used in classroom settings: (i) the educator conducting the activity, (ii) the classroom teacher's behaviors, and (iii) students' behaviors. The outcomes of modeling activities offer important insight in helping educators identify potential problems while attempting to integrate them into teaching environments. In classes in which such activities are performed, determining the trends, behaviors, and opinions of students, teachers, and those PTs who have received MM training, including training about the approaches used to perform such activities will aid in making in-class adaptations of modeling activities more effective. It is therefore postulated that knowing how teach and integrate mathematical teaching and, even more importantly, MM activities into teaching environments will have a positive impact on teachers' future practice.

The current study, therefore, seeks to identify primary school mathematics PTs' opinions about performing MM activities. To this end, answers to the following question were solicited.

What are PTs' views on performing MM activities?

3. Method

3.1. *Research Design*

This study follows a case study methodology. Being qualitative in nature, case studies seek to gain further data on and a better understanding of a specific individual, group, event, or organization (Patton, 2014). Accordingly, the case investigated in the current study is PTs' opinions about creating modeling activities.

3.2. *Participants*

Participants consisted of 18 middle school mathematics PTs in their fourth year of study in the Faculty of Education of a state university located in Turkey's Marmara Region. Criterion sampling, a type of purposeful sampling, was used to select participants. As such, all volunteer PTs in their fourth year of undergraduate study enrolled in the MM elective class participated in the study.

Moreover, all participants had previously completed several content education and research methods classes like Analysis, Special Education Methods, and Scientific Research Methods.

3.3. *Research Process*

This study was conducted as part of the participants' undergraduate *MM* class. PTs were first provided theoretical information on models, the definition of modeling and MM, areas of use, basic components, different MM perspectives, activities and the modeling process, and in-class applications of MM. During this process, PTs also completed their own literature review in groups in which they investigated the solutions of different modeling activities. PTs were additionally asked to use EDMODO, an internet-based classroom management program designed for educational use, to report their opinions about the subjects and activities they had covered during lessons to the researchers at the conclusion of every lesson. Accordingly, PTs wrote what they thought about the activities they had completed at the end of each class in the form of a diary, which they then shared with the researchers. These diaries were read by the researchers every week, and feedback was provided to the diaries' owners if necessary. This way, participants were able to obtain sufficient theoretical knowledge about MM as well as ample experience solving related activities.

Traveling to the schools where they performed their practicum together with their fellow group mates, PTs video recorded the modeling activity that they had performed in class, which was shared with the other students in their modeling class. These activities consisted of activities thought to build relationships with middle school students' real lives. PTs realized these activities in groups of four to five individuals. Additionally, the middle school students' own mathematics teachers were included in three of the four activities conducted. Video recordings of the activities performed over a four-week period by four separate groups were watched in PTs' MM class, which served to convey their in-class experiences with their classmates. Every week, PTs offered their opinions and assessments of the videos in question.

Before performing any activity in a classroom setting, PTs chose several activities from a larger pool of activities. Therefore, different factors (e.g., how appropriate activities were for students' levels, activities' ability to attract students' interest, scheduling) related to the activities performed in PTs' MM class were taken into consideration beginning from the very start of the semester. Subsequently, documents importing on the significance of making an appropriate lesson plan and on the necessity of devising an effective division of labor during preplanning were submitted to the researcher during the MM class. PTs' preplanning documents were evaluated based on several criteria, including

(i) how many individuals were to be included in groups based on class size, (ii) what concepts were mentioned during warm-up activities prior to moving onto the activity's problem, (iii) how much time was to be given to solve problems, and (iv) the order that groups were to present their solutions. PTs were also provided feedback by the researcher. In order to prepare the participants for the activity, a number of adjustments were made, like reducing the number of in-class groups, providing participants with sufficient preliminary information about the activities to be performed, and increasing the amount of time they had to solve problems. Furthermore, PTs were also asked to include detailed explanations about the warm-up phase of the lesson, about how they had moved on from the activity's introduction to the actual problem itself, about their experiences with other groups and offering guidance, and about how they were planning to share various in-class responsibilities, like obtaining a video camera for the class. This was done to prevent any potential chaos from occurring during class.

PTs showed the video recordings of the activities they had performed with their peers in their MM class. The students were asked to watch the videos and listen to their fellow classmates describe their experiences while conducting these activities. After sharing videos and experiences, students participated in class-wide discussions in which the activities performed, the PTs conducting them, the students involved, the classroom environment, and the classroom teacher's attitude were all addressed from various angles. PTs listened to each other as they shared their experiences and even offered each other feedback based on the MM training they had received. Immediately following this, the researchers then used a semi-structured observation form to record what each PT thought about the video they had watched one day earlier, which was then uploaded to EDMODO. This was repeated for four weeks. Figure 1 gives a general overview of the research process.

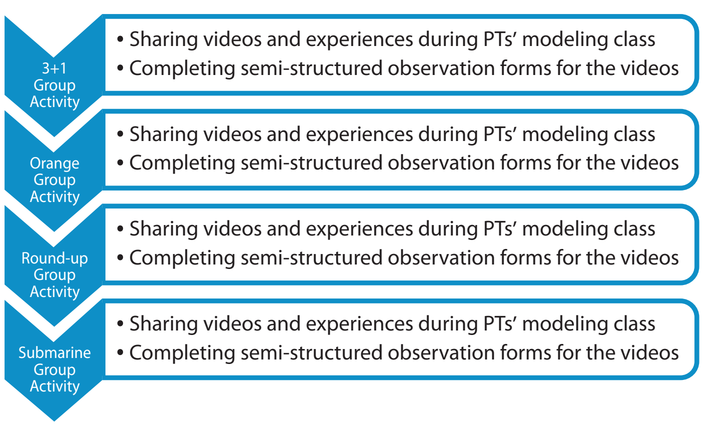


Figure 1. General overview of the steps followed during research

3.4. *Data Collection*

The following means were used to collect data for the current study: (i) lesson and execution plans for the activities PTs were to conduct, (ii) semi-structured observation forms, and (iii) notes taken by the researcher. Table II summarizes the activities that participants chose and implemented.

TABLE II
Activities implemented by participants

Activity Name	*Name of the Group Executing the Activity*	*Concept Addressed by the Activity*	*The reasons why teacher candidates choose these activities*
Big Head	3+1	Ratio and proportion	Attractive and suitable for students' level
Largest Shoe Size in the World	Orange	Ratio and proportion	Attractive and suitable for students' level
Horse Race	Group Round-up	Probability	The subject of probability is difficult for students. Students can understand the importance of knowing probability.
Bales of Hay	Submarine	Pythagorean Relation – Relationship between Diameter and Radius	Attractive and enjoyable

The activities in table II are briefly given below.

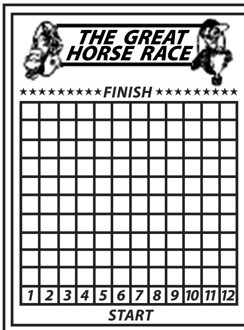
What would be the size of the statue if it showed Adenauer from head to toe at the same scale?

Figure 2. Big Head Activity adapted from Herget, Jahnke & Kroll (2001)

The person who fits this giant shoe must have enormous feet!

Antal Annus, a 73-year-old shoemaker from the Hungarian village of Csanédapáca, is depicted here, proudly presenling his hitherto most impressive "creation" To this very day, we still do not know whether he really made the shoe for one of his customers.

Figure 3. Largest Shoe Size in the World Activity (Herget, 2000)



The Great Horse Race Rules

1. Put the horses on their starting positions, 1 to 12.
2. Each player chooses a different horse.
   If there are only a few payers, then each can choose two or three horses. The remaining horses are still in the race but no one owns them.
3. Roll two dice and add the scores.
4. The horse with that number moves one square forward.
5. The first horse past the finish wins.

Figure 4. Horce Race Activity (Swan, Turner, Yoon & Muller, 2007)

In the figure below, there are 5 hay bales at the bottom. Hay bales is disposed to decrease towards the upper. Accordingly, find the height of the whole stack.

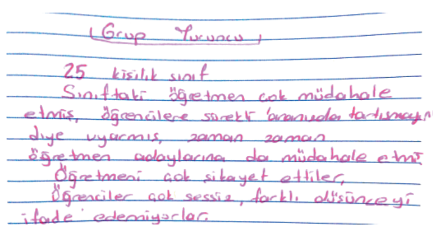
Figure 5. Bales of Hay Activity (Blum & Leiß, 2007)

Generally, pre-service teachers stated that they paid attention to the fact that the activities were easy and interesting because the students were not accustomed to such activities. They thought that it would be fun for students to compare their own body measurements in activities related to ratio-proportion. In the activity related to probability, they reported that the students had difficulties in probability, so they chose an activity that explained the importance of knowing probability.

PTs were given semi-structured observation forms after watching videos. Some of the questions included in these forms are:

– Was the activity effective in teaching the mathematical concept at hand?
– What are your observations of the classroom atmosphere?
– What do you think about (e.g., problems encountered, recommendations) the PT performing the activity?

Additionally, looked at research took her own notes while PTs' videos were being screened and while they shared their experiences with their peers. Figure 6 offers an example of such notes.



Group Orange
Class for 25 students
teacher has intervened a lot
teacher, constantly 'do not argue between you' he warned.
The students are very quiet, can not express themselves.

Figure 6. Example of notes taken by the researcher

3.5. *Data Analysis*

The data generated from this study were subject to content analysis. After watching the video recordings of activities, all the observation notes made by PTs were collected and transferred to a qualitative data analysis program. Then, all statements were coded and appropriate categories were made. These categories were themselves grouped under two headings, namely *Activity-Student-Teacher* and *Preservice Teachers.* Table III depicts a short section of the data analyzed. The themes addressed by PTs were divided into various categories and subcategories. Specifically, the subcategories included under the category *Classroom Environment* were: (i) active participation, (ii) communication-collaboration, and (iii) modeling process whereas the subcategories included under the category *Difficulties Experienced* were: (i) understanding-modeling, (ii) group work, and (iii) expression-assessment.

TABLE III

Example of PT's Opinions regarding Middle School Students and codes used by theme

Category	*Sub-category*	*Statements*
Classroom Environment	Active participation	P1: By having students work in groups, having them collaborate, and providing them venues where their self-confidence would be bolstered by valuing the ideas they came up with while trying to solve problems made positive contributions to their affective qualities. (Activity 1)
		P2: All groups worked hard to solve the problems at hand. Students who solved the problem quickly were keen to explain their solutions to their peers and did so with enthusiasm. When a student who fully understands the solution was chosen at random to stand up and explain his answer, the other students listened attentively to understand him. Students continued to explain their solutions even after the bell had rung, thereby foregoing their break time. (Activity 2)
	Communication - collaboration	P14: As a result of this activity, students were given the opportunity to share their thoughts, make educated guesses, and discuss their opinions as groups, which, similar to the activity that immediately preceded it, allowed them to develop their discussion skills and provided them an opportunity to showcase their collaboration skills. They were also able to use their prior mathematics knowledge. All of these were the result of long discussions and exchanges of ideas.
	Modeling process	P2: At first, there were some who designed and worked off of a case model. They drew the statue and reference point when they were explaining their solutions on the board. They thought about the ratio between the head and the entire body. Many groups used the height of the child on top of the statue as a starting point to estimate the size of the statue's head. They then estimated how big a person's head was compared to his body and used this to estimate the height of the entire statue. One group measured the child's height with a pen cap and another used his finger to measure it. One group used the street lamp as a reference point and determined the ratio between it and the statue's height. Some used the bust of Ataturk as a reference point to solve the problem. They all used appropriate methods to conduct their modeling process. (Activity 1)

Difficulties experienced	Understanding - modeling	P6: All of the students got hung up on the fact that the shoe maker was 73 years old, thinking that this information was necessary. This is because they have been trained to think that they need to use any mathematical information, even if it is simply extra, unrelated information. (Activity 2)
	Group work	P5: The first two groups were listened to and were able to share their ideas in a respectful and understanding manner. Although these groups were quite successful in completing their modeling activity and in providing solutions to the problem, some groups simply accepted the ideas of their speaker or of the 'group leader' without question and, instead of working as a group, simply rejected the otherwise bright ideas brought up by the other members in their group. When it came time for them to give their presentations, it was clear that they had tried to complete the activity without actually working in a group. (Activity 3)
	Expression - assessment	P6: The writing portion of this part was very problematic. Although I had repeated several times that they needed to make a report of the information I had given them multiple times before, the activity was still very difficult. Though some of the students were able to write it down on paper very well, when they were called to the board, it was as if they retained nothing and had completely lost their self-confidence. I think that they had difficulties because I had not provided them ample time or venues to express themselves. (Activity 3)

3.6. *Trustworthiness of Research*

One means to increase the trustworthiness of qualitative studies is the triangulation technique (Denzin, 1978). Accordingly, this study made use of PTs' lesson and execution plans, semi-structured observation forms, and field notes to collect data. To further increase the data's trustworthiness, peer assessments were employed. According to Guba and Lincoln (1994), peer assessment constitutes an external control mechanism for trustworthiness. That being the case, the themes and categories created by the researchers were sent to a researcher with a PhD in mathematics education and whose comments were used to make amendments to six subcategories. For example, while the category *Modeling Process* had been considered a theme related to activities, it was decided that it should be considered a student-related theme because it reflected the classroom environment and students' behaviors. Furthermore, we used participant confirmation to determine whether studies' findings accurately reflected their own opinions (Merriam, 2013). For this, the researchers conducted data analyses and, after the study had been completed, were reported and all data collected. These reports were then sent to participants, who were asked to indicate whether the analyses accurately reflected their own opinions and the research process.

4. Findings

Following the data's analysis, PTs' opinions regarding the execution of modeling activities were divided into four main themes. Findings related to these themes are presented below:

4.1. *Opinions regarding Activities*

Figure 7 summarizes participants' opinions on the activities conducted in class.

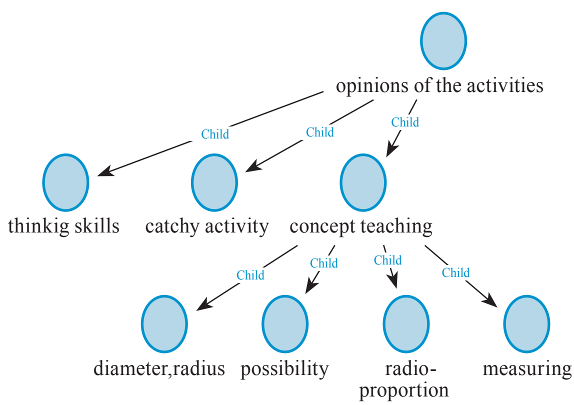


Figure 7. Participants' opinions regarding activities

Figure 7 illustrates how participants' opinions about activities were grouped into four separate categories. This chart indicates that PTs considered activities not only to increase various thinking skills of students and to be interesting, and could be used in teaching the concepts at hand.

Regarding *Thinking Skills*, PTs discussed how they thought the activities had influenced students' analytical, creative, and mathematical thinking skills, their sense of numbers, and their ability to observe relationships with real-life events and to consider issues from different perspectives. 'I think it helps to understand the subject better and use it in daily life' (Rukiye- pre-service teacher- for big head activity). In general, since all the comments made by PTs indicated that the activities attracted students' attention, it may be concluded that activities in the form of games or that are related to real life attract students' attention.

'The activity was interesting. Most of the groups were willing to figure it out. It felt different because they had not done such activities before and they did not do group work. Several groups discussed among themselves and found good results and expressed themselves well on the board. They discovered estimation skills and how to use ratio in such a problem' (Didem-pre-service teacher-for Largest Shoe Size in the World activity'

With regard to *Concept Teaching*, PTs discussed which activities could be used to teach which concepts. Appropriate answers were given to the designated target objectives in the first three activities; however, although the target objective in the final activity dealt with the Pythagorean relation, no PT mentioned it.

'Even though the students understood the concept of number sense, the main purpose was not to give them the concept. Our friends should have given this concept as additional information. They put too much emphasis on it, and from this point of view, the students perceived this concept of number sense as if it was a subject covered in a mathematics lesson. While the concept to be given in this activity is the Pythagorean relation, no student in the class has even used this concept' (Nurseda- pre-service teacher-for horce race activity)

In addition to stating that activities could be used to teach the concepts in question, PTs also articulated that activities actually had a greater impact on the other categories composing this theme than they did on concept teaching.

'We saw that the students tried to reach a solution by estimating the length of the shoe given in the picture, the shoulder width of the man holding the shoe, the glasses, the waist width and proportioning it with the shoe. Thanks to this activity, students put forward their thoughts by making assumptions like the previous activity, contributed to their discussion skills by discussing their thoughts in the group, had the opportunity to demonstrate their group work skills, and used their previous mathematical knowledge. Here they gained a lot of skills rather than an activity on 'ratio-proportion. This part was more important' (Seda- pre-service teacher- for Largest Shoe Size in the World activity').

4.2. *Opinions regarding Preservice Teachers*

Figure 8 depicts what participants thought about their peers after having watched video recordings of them conducting activities and listening to them share their experiences.

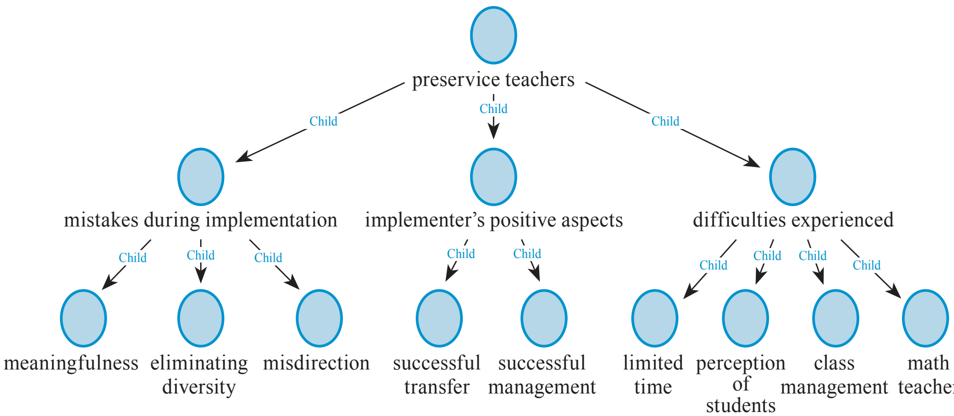
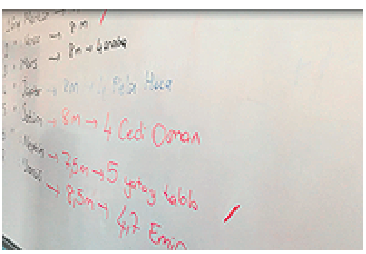


Figure 8. PTs Opinions regarding their Peers' Execution of Activities

Figure 8 indicates that PTs' opinions regarding their peers were divided into three separate categories. Accordingly, PTs' opinions regarding their peers' behaviors during activities considered to be improper were entered in the subcategory *Mistakes during Implementation*. Those opinions on the behaviors PTs' deemed to be constructive were entered in the subcategory *Implementer's Positive Aspects*. Finally, statements made regarding the most difficult situations encountered during activities' implementation were entered in the subcategory *Difficulties Experienced*.

Accordingly, the mistakes that PTs made during activities' execution were further grouped into three separate subcategories, namely (i) meaningfulness, (ii) eliminating diversity, and (iii) misdirection. With regard to M*eaningfulness*, PTs stated that additional preliminary information could have been given or more concrete materials could have been used to make the activity more meaningful for students and that it was necessary to spend more time on the preplanning phase instead of moving on to the actual problem, as that would help students better understand problems. With regard to *Eliminating Diversity*, statements existed indicating that student groups attempting to use different means to address the problem were interfered with and that groups were persuaded to solve the problem the same way that PTs had. For example, an examination of the third activity (i.e., Horse Race) reveals that one group of students thought it was necessary to use dice in order to solve the problem in the activity and, accordingly, started to make a die out of paper. Upon seeing this, the PTs orchestrating the activity advised students that dice were a waste of time and made several other recommendations instead. During the fourth activity (i.e., Bales of Hay), students modeled the positions of the bales of hay without actually looking at them and made a link between their conclusion and objects that existed in their own environments (Figure 3). After all of the groups had finished their presentations, one student mentioned that it was necessary to consider the positions of the bales and that it was therefore necessary to make another, entirely different model. However, the PTs conducting the activity did not ask the students to find another solution after this student had realized what needed to be done and instead ended the lesson. Consequently, the PTs watching the video recording of this activity criticized its implementers, stating that it is absolutely necessary for different ideas like the one in question to be given a place in classes.

With regard to *Misdirection*, after PTs had watched video recordings of the fourth activity (i.e., bales of hay), they articulated that some of their fellow PTs had misguided students and that they had not achieved their stated goal. At the beginning of the activity's execution, PTs' made comments on the sense of numbers during the warm-up activity that caused students to address the problem using various forms of estimation instead of with mathematical methods. Students were unable to construct an accurate model of the bales of hay by placing them on top of each other and attempted instead to solve the problem building off of relationships with real-life objects and people's heights. Figure 6 shows the solutions reached by all the groups in the class.



Group 1. Mercury → 8m
Group 2. Venus → 8m
Group 3. Anthem → 8m → 4 cars
Group 4. Jupiter → 8m → Pelin (their teacher)
Group 5. Saturn → 8m → 4 Cedi Osman
Group 6. Neptune → 7,5m → 5 Table
Group 7. Uranus → 8,5m → 4,7 Emir (their friend)

Figure 9. Example of Activity 4

Figure 9 illustrates that students attempted to associate the total height of the bales of hay with people and objects found in their own environments.

In addition to offering constructive criticism to their peers, PTs also stated that some of the activities were effectively executed, and such opinions were included in the category *Implementer's Positive Aspects.* Specifically, PTs successfully described all that had happened in the classroom, collaborated appropriately with their fellow PTs, and were able to manage the class effectively. These opinions may be considered evidence that PTs came prepared for their lesson, were enthusiastic, and successfully completed the activities.

The category *Difficulties Experienced* mostly includes statements pertaining to PTs' experiences while conducting activities. Several PTs stated that they had insufficient time to complete some of the activities when responding to their peers' criticism toward their implementation of activities.

‘ [..]We were very happy that a student finally noticed this. But we couldn’t say yes to the 30 people who enthusiastically held the event there. That would be to discourage them. That’s why we said your friends made the right point. But we said that our request from them is an estimated value, not a definite result. I do not think that we discouraged the student who realized this situation, and extinguished him. We congratulated and thanked him for noticing this. Our friends said that we should direct students to the right solution. Yes, then we could have achieved a tremendous result. But we didn’t have much time. We could only do so much in a limited time’ (Feyza-pre-service teacher - for bales of hay activity).

In fact, several articulated that the fact students were unfamiliar with this type of activities was an obstacle to their smooth progression and that the limited amount of time caused them stress. With regard to C*lassroom Management*, statements indicated that students’ inability to fully comprehend the activity and other factors like classroom teachers’ attitudes caused PTs severe difficulties in managing the class. That said, PTs stated that they would have faced even more difficulties had they been required to work on their own instead of working in groups of 4-5. Finally, the classroom teachers working with PTs were more likely to exhibit behaviors that hindered the activity’s objectives from being accomplished than otherwise.

4.3. *Opinions regarding Students*

PTs’ opinions regarding the middle school students with whom activities were conducted were divided into two separate categories, as illustrated in Figure 10.

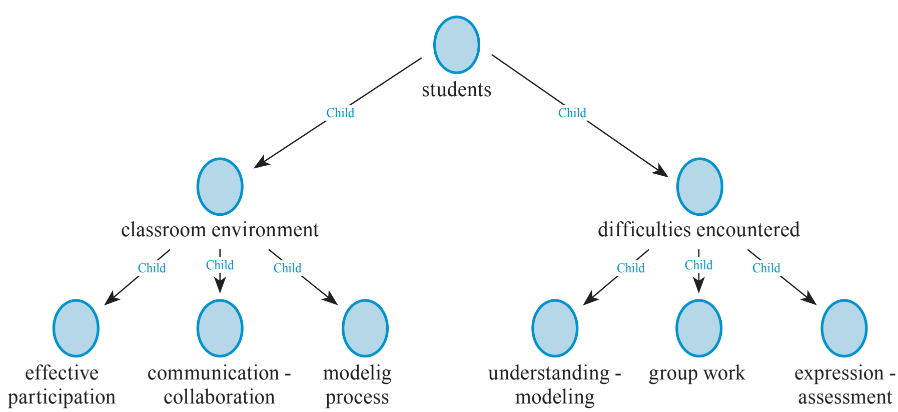


Figure 10. Participants’ opinions regarding students

Figure 10 reveals that participants’ opinions regarding students are divided into two separate categories, namely *Classroom Environment* and *Difficulties Encountered*. Under the former category are the subcategories *effective* *participation*, *communication-collaboration*, and *modeling process* whereas under the latter category are the subcategories *understanding-modeling*, *group work*, and *expression-assessment.*

PTs' statements indicating that students had approached the activity with a positive mindset and had thought over and discussed the problem for a significant amount of time in order to find an appropriate solution were included in the subcategory *effective participation*. Included in the subcategory *communication-collaboration* are statements by PTs indicating that students had discussed their solutions in groups, had communicated with others, and in short, had shared their ideas with their peers. In the subcategory *modeling process* are statements made by PTs indicating that students used math first to construct and then to solve the problem after understanding what was being asked of them. During all of this, however, PTs determined certain areas in which students struggled. One such area was *understanding-modeling*, in which students had difficulties both in understanding the activity and in constructing relationshipships between real life and mathematics. In an effort to remedy this, PTs resorted to numerous strategies to guide students. Moreover, since students were unfamiliar with group work etiquette, PTs articulated that the students struggled to work together in groups in an effective manner. PTs' statements describing such difficulties were included in the category *group work*. Finaly, although several students reached a specific conclusion, they were unable to express themselves in a convincing manner. PTs' statements indicating that students had difficulties deciding as to whether their conclusions were accurate or not were included in the category *expression-assessment*.

4.4. *Opinions regarding Teachers*

PTs' opinions regarding the mathematics teachers partaking in the activities with them were included in the theme *Mathematics Teacher.* The three categories included in this theme are illustrated below in Figure 11.

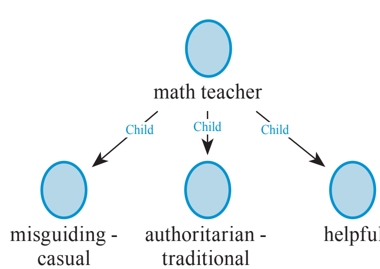


Figure 11. Preservice Teachers' Opinions regarding the Mathematics Teachers Participating in Modeling Activities

Figure 11 depicts the subcategories into which PTs' opinions regarding classroom teachers were divided, namely: *misguiding-casual*, authoritarian, and *helpful*. These categories included opinions of different teachers. For example, PTs' comments on the teacher participating in the first activity (i.e., Big Head) were included under the subcategory *misguiding-casual*. PTs stated that they had difficulty maintaining control over the class as a result of this particular teacher's overly casual attitude. The same teacher was also stated to have given hints to students without allowing them the opportunity to think about the problem for themselves. 'The teacher wanted to help the students. But he misdirected. There was a more casual classroom because of the classroom teacher. They saw our friends who went to the practice not as teachers, but as older sisters/brothers' (Sude-preservice teacher-for big head activity).

Under the subcategory *authoritarian-traditional* were comments made about the second activity (i.e., Shoe Number). The teacher who participated in this activity exhibited a very authoritarian attitude throughout the entire activity, which negatively affected the entire course of the activity 'The groups were supposed to discuss together, and it was normal for noise to come out during this time. However, the teacher's warning at every sound prevented the students from discussing and arguing' (Nurseda-preservice teacher).

The third activity (i.e., Horse Race) included PTs' statements about a teacher who would support them whenever they asked. This teacher would exchange ideas with the PTs about forming groups to ensure that they were not homogeneous and about which students should present their group's findings to the class. Finally, since no mathematics teacher participated in the fourth activity (i.e., Bales of Hay), no category was created for it.

'The teacher behaved exactly as he should have. He helped our friends with classroom silence and control. In addition, the students did not direct them while doing the activity. It is clear that he is much better than the teacher standing at the head of our friends last week' (Şevval-preservice teacher).

5. Discussion

In this study, opinions regarding middle school mathematics PTs' execution of MM activities were discussed. As such, PTs' opinions were categorized under four themes.

The first theme created was *Activities*, which indicated how the format of activities impacted students. As a result, four separate categories were created. All of these categories consist of specific features emphasized in modeling activities. Lesh and Doerr (2003), as stated above, have emphasized there to be four separate dimensions that students working with modeling activities learn, namely behavioral (basic facts and skills), process (mental habits not linked to a specific mathematical structure), affective (attitude, beliefs, and emotions), and cognitive dimensions. As such, the current study shows that from among the categories included under *Activities*, students' affective dimension is influenced by *interestingness* whereas their behavioral dimension is influenced by *concept teaching*. Statements included in the category *thinking skills*, however, describe the structure that Lesh and Doerr (2003) call mathematical thinking in MM. This is because mathematical thinking gives greater precedence to making interpretations and giving explanations than to simply performing arithmetical calculations. As such, since mathematical thinking aims for students to construct conceptual systems instead of simply finding the correct answer, these activities, when performed in classroom settings, are observed to facilitate the very thinking system that MM seeks to instill in students. Furthermore, PTs' statements revealed that these concepts' being associated with and organized in light of real-life events instead of simply facilitating students' superficial understanding of the concepts sought to be taught led to significant development in their mathematical thinking skills. An examination of the literature reveals that unlike the prevailing claim in the constructivist approach that deems it necessary for knowledge to be constructed/taught from scratch, the MM approach followed in mathematics education emphasizes that it is more important for individuals to organize a system or construct a relationship based on what they already know than to construct all the mathematical systems and rules. In other words, while the constructivist approach considers it necessary for individuals/students to discover new ideas and concepts emerging as a result of the ever-developing conceptual systems in their minds, MM aims for individuals/students to understand and conceptualize patterns and systems outside of their own minds instead of focusing on what the term discovery actually means (Lesh & Doerr, 2003). Accordingly, the fact that activities practice concepts already learned, organizational skills, and making associations instead of simply teaching concepts resembles the MM approach. According to the current study's findings, PTs stated that they had realized this fact while executing the activities in class. However, it is known that there are 2 main features of mathematical modeling activity. (school-based modeling activities offer a window beyond the school, (2) offer a window into the world beyond the school on the use of school-based mathematics in practical situations (Vos, Hernandez-Martinez, & Frejd, 2020). Therefore, the fact that the activities develop thinking skills more than the gains is an indication that the activities have a structure that can positively affect the future lives of the students.

Regarding student-related categories, the fact that students actively and enthusiastically participated in activities and were satisfied with the classroom environment is clear evidence that these activities were related to real life.

However, the fact that students were unable to work in collaborative learning environments or to perform self-evaluations indicates that they were unfamiliar not only with modeling activities but also with other activities based on a constructivist philosophy. This finding is valid both in Turkey and other countries. The problems that students encounter are generally only distantly related to the actual modeling process, and modeling activities play a minimal role in mathematics teaching (Blum & Borromeo, 2009; Doerr & English, 2003; Pollak, 2003; Zawojewski, 2010). In themes related to students, participants stated that in addition to students' experiencing difficulties as a result of their unfamiliarity with the activities, the fact that students exhibited positive attitudes toward the activities indicated that they could very well complete these activities in the proper learning environment and if correct guidance were given. In fact, several studies have revealed that in the event that MM is taught properly, students are certainly able to learn how to construct and use such models (Maaß 2006; Biccard & Wessels 2011; Blum & Leiß 2007).

Regarding *Mistakes during Implementation*, PTs stated that they had completed the meaning-making phase of the activity too quickly and had moved on to the actual problem before students had completely understood the context. PTs completed the activities within 1-2 class periods. As such, instead of restricting activities to 40-45 minute periods, activities can be completed within a more flexible time period in which all ideas are given due consideration. This may provide opportunities for the teacher to complete every stage of the modeling process in greater detail and care.

Regarding themes pertaining to teachers participating in activities alongside PTs, the two teachers involved in the first two activities (i.e., Big Head, Shoe Size) exhibited behaviors inconsistent with those recommended in the MM approach. Teachers' interference in the process made the activity even more difficult. A look at the names of the categories that the behaviors of these two teachers were included (i.e., misguiding-casual and authoritarian-traditional) revealed that they exhibited contradictory behaviors (i.e., casual-authoritarian). In other words, teachers' mix of casual and authoritarian attitudes caused students to have difficulties completing activities. In MM, similar to constructivism, the teacher is considered to be an advisor and a guide who, using questions, directs students to express their own ideas (Burkhardt, 2006). Because the teacher did not interfere in the activity's execution in the third activity, however, no problems were experienced. The teachers participating in the first two activities did not exhibit behaviors considered appropriate for MM or in line with a constructivist philosophy. That said, regardless of how much information they were given prior to the activity, it is possible that teachers' interference in the process stemmed from their not wanting their own teaching to be perceived poorly in the video recordings made in class.

A lasting balance between (minimum) teacher guidance and (maximum) student independence is essential for quality teaching (Blum & Borromeo, 2009) Similar studies reveal that teachers unconsciously tended to direct students toward their own ways of solving problems and that this stemmed from their lack of information regarding the scope of their duties (Borromeo & Blum, 2009; Blum, 2015). Likewise, the current study revealed that teachers needed additional knowledge about their classroom roles and the type of teaching that was to be used.

In addition to all of the above, although PTs had received MM training, they were still found to have some gaps in some of their knowledge and skills, which caused them to receive criticism from their peers. An examination of PTs' *Mistakes during Implementation* revealed that both PTs and students hastened to reach the expected solution as soon as possible. In other words, as described in the modeling steps (Borromeo, 2006), PTs attempted to persuade students to accept their own plans instead of those that each of them had devised on their own. Instead of encouraging them to understand their own thinking processes, students were, in a manner of speaking, asked simply to imitate their teachers. In a similar vein, Yu and Change (2009) examined the studies on classroom environment conducted in Taiwan and found that the majority of mathematics teachers continued to teach the same way they had learned, that classes generally consisted of teachers transferring their own knowledge to students, and that the main reason that such activities were not completed in class was because teachers felt no unease in completing the school curriculum and considered standardized tests to be an appropriate indicator of academic achievement.

The current study indicates that although teachers had learned what kind of profile they needed to adopt during the modeling training that they had received prior to executing the activity, they had difficulties in putting it into practice. Reasons for this included the amount of time allotted for activities, discomfort caused by excessive student noise, and the need for the activity to be successfully completed. Using Lesh and Doerr's (2003) four main goals in mathematics education to interpret this finding, it appears that both behavioral (e.g., input-output, reaching one's goals) and affective goals (e.g., students fear of liking activities) eclipsed the cognitive process. Accordingly, improving the cognitive aspect of activities has been stated to facilitate teaching mathematics and to pave the way for students to make greater process in successfully completing mathematics-related objectives compared to other targets (Stigler & Hiebert, 2004). Keeping in mind that the most beneficial act able to be done in the classroom environment is to nurture an environment in which high cognitive goals are held and in which students are provided with ample opportunities to practice and develop their reasoning skills (Stein, Smith, Henningsen, & Silver, 2009), those aspects that impede the acquisition of cognitive goals may be eliminated and ideal activities may be conducted. This will, in turn, allow all variables affecting in-school learning (e.g., curricular targets, examination types, classroom environment, and both class length and times) to be restructured in such a way that they focus on students' cognitive development.

6. Conclusion

In this study, opinions of PTs on carrying out MM activities were determined. Accordingly, PTs reported that the activities revealed students' different thinking skills. pre-service teachers who received modeling training tried to provide an appropriate learning environment in practice. They identified some problems in their interventions with students and attributed these problems to some factors such as limited time, readiness of students, and attitudes of teachers. They stated that the students liked modeling activities very much, they approached them with interest and curiosity, but they did not know exactly how to behave. They also stated that the activities revealed the different thinking skills of the students. On the other hand, they reported that the teachers made unnecessary interventions to the students and this situation negatively affected the modeling process.

7. Research Limitations and Recommendations for Future Research

Using PTs' own statements and opinions, this study attempted to reveal how MM activities were executed. Future studies can include the opinions of students and fully-licensed teachers to increase the overall scope of the research.

In the current study, however, the process by which MM activities were executed was laid out and the conclusions were discussed in light of all of the system components influencing activities' execution (i.e., curriculum, target acquisitions, teaching methods, assessment, and classroom environment). Indeed, teachers constitute the most potent element in conducing MM activities in schools. Consequently, this study revealed that regardless of the difficulties faced by students, they approached activities positively and were enthusiastic to participate in them. That said, however, both teachers and PTs exhibited normative, restrictive behaviors during activities' execution. PTs justified these behaviors citing time limits, classroom management, and not wanting to discourage students. This study further revealed that further investigation into mathematics education is required. Building off the concept of mathematics literacy and considering that an individual's ability to use mathematics to solve real-life problems is more important than simply learning mathematical concepts and terminology, it is necessary to give greater importance to studies focusing completely on the various components making up the system, namely the curriculum, target acquisitions, assessment and evaluation, teacher education, and learning environments (Blum, 2015), in order for modeling activities to be successfully implemented.

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