EDITORIAL

23 DESAFÍOS MATEMÁTICOS
"*DEBEMOS SABER. LO SABREMOS*"

23 MATHEMATICAL CHALLENGES
"*WIR MÜSSEN WISSEN. WIR WERDEN WISSEN*"

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Recientemente se publicó en el sitio https://lims.ac.uk/23-mathematical-challenges/ una información que considero relevante para nuestro campo y su futuro, la Matemática Educativa, la nota fue titulada "23 Mathematical challenges".

Esta lista fue compilada por el *London Institute for Mathematical Sciences* en virtud del lanzamiento de la nueva agencia científica británica ARIA (*Advanced Research and Inventions Agency*), y su objetivo es abordar los problemas más difíciles de nuestro tiempo.

A principios del siglo XX, específicamente en el año 1900, el Prof. David Hilbert hizo pública una lista de 23 problemas matemáticos importantes. Desde entonces, 17 problemas se han resuelto total o parcialmente y, la lista ha sido seminal para las Matemáticas.

Se espera que los nuevos 23 desafíos matemáticos interesen durante el siglo XXI, sólo que ahora fueron planteados en términos transdisciplinares con algunos ciertos criterios de demarcación. Esto es un sello de nuestros tiempos, pues trata de un cambio de paradigma que asume, en un sentido amplio, la célebre frase de Galileo respecto a las matemáticas, son el lenguaje de la ciencia y los avances científicos más transformadores dependen de ellas. La teoría de números, se cita como ejemplo, está en la base de la criptografía moderna.

Para conformar dicha lista, el *London Institute for Mathematical Sciences* organizó un simposio entre físicos y matemáticos. En los preparativos del encuentro, se les solicitó a 23 líderes mundiales en la investigación solicitándoles sus contribuciones. Se reunieron, de este modo, más de 100 desafíos que luego redujeron a 23. Según narran en su nota, se siguieron tres criterios para elegir la lista de desafíos matemáticos. 1. En primer lugar, se favorecieron los problemas

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cuya investigación tenga altas probabilidades de resultar fructífera (*demarcación de factibilidad*), incluso si no conduce a una solución real. 2. En segundo lugar, buscaron equilibrios entre problemas concretos con respuesta definitiva y desafíos especulativos planteados libremente. Estos tienden a ubicarse en campos establecidos y aquellos otros que recién emergen (*demarcación de diversidad*). 3. En tercer lugar, se procuró equilibrar la lista en toda la gama de ciencias matemáticas (*demarcación de cobertura*).

Para prepararse ante estos desafíos, la ARIA bien podría considerar el epitafio de Hilbert, cuyas palabras en su tumba del matemático de Göttingen dicen: "Wir müssen wissen. Wir werden wissen". En castellano esto sería: "Debemos saber. Lo sabremos".

Hacemos un llamado a la comunidad de Matemática Educativa, para organizar espacios de reflexión sobre nuestros propios desafíos, que sin desatender a la diversidad teórica y metodológica que nos caracteriza, consideren a su vez la dimensión regional y cultural en la que se lleva a cabo la tarea educativa.

Nos parece que la lista de desafíos matemáticos se intersecta fuertemente con nuestro quehacer, pero obviamente no lo cubre, pues algunas de las áreas emergentes en nuestro campo son propias de la dinámica de enseñanza y de aprendizaje de saberes escolares y corresponden también a una amplia gama de temáticas de corriente sociocultural, como las denominadas construcción social. Estas últimas han ido modificando la clásica centración en las estructuras cognoscitivas del sujeto, para explicar al aprendizaje matemático dentro de la psicología del pensamiento, para incluirlas como el resultado de la vida en sociedad.

Dejaremos en su idioma original, para mayor fidelidad, los citados desafíos esperando con ellos, la construcción de desafíos propios de la Matemática Educativa para Latinoamérica.

1. *Theory of everything*

We lack a single theory that describes the universe. Gravity, described by general relativity, is not consistent with our quantum field theory of the other three forces. Will this be resolved by string theory, loop quantum gravity, or something new? What are the testable consequences of such a theory, which is beyond the limit of human experimentation?

2. *Riemann hypothesis*

Attempts to settle the Riemann hypothesis have inspired whole new branches of mathematics. For example, the Riemann zeta function is the simplest kind of L-function, and these seem to play a role in modern mathematics similar to polynomials in ancient mathematics. What new concepts are needed to resolve this most important of open problems?

3. *Thermodynamics of life*

According to Darwin's theory, evolution is the result of mutation, selection and inheritance. But from a physics perspective, we don't understand how life got started in the first place. What is the thermodynamic basis for emergent self-replication and adaptation, of which biology is just one instance? Can it be used to create digital artificial life?

4. *The structure of innovation*

Despite advances in our understanding of evolution, what drives innovation remains elusive. Technological innovation operates in an expanding space of building blocks, in which combinations of technologies become new technologies. Can we characterise innovation in a mathematical way, so that we can predict it and influence it through interventions?

5. *Physics of self-assembly*

Self-assembly is how proteins fold, snowflakes form and viruses assemble. It can be used to manufacture complex and nanoscale objects at low cost. Because it is a physical embodiment of computation, it is deeply linked with decidability. Can statistical physics be combined with computability theory to build a comprehensive theory of self-assembly?

6. *Cosmological constant*

Only a small fraction of the observable universe is made up of known matter. The majority is conjectured to be dark matter and dark energy, for which there is no consensus in explanation. Why does the zero-point energy of the quantum vacuum not cause a large cosmological constant? What cancels it out? Is new fundamental physics needed to reformulate gravity?

7. *Langlands Program*

There is evidence of a grand unified theory for mathematics, called the Langlands Program. It seeks to relate automorphic forms in geometry and number theory to representation theory in algebra. Wiles' proof of Fermat's Last Theorem can be viewed as just one instance of it. How can we advance and extend this Program, and what fruit will it bear when we do?

8. *Intelligent AI*

Far from approaching artificial general intelligence, AI has not progressed beyond high-dimensional curve-fitting. What mathematical insights could lead to more intelligent AI, such as causal reasoning, functional modules or a representation of the environment? Are there fundamental limits to AI, and what might this tell us about human intelligence?

9. *Repairable instead of robust*

To be sure of success in the face of uncertainty, we make plans that can cope with the unexpected. One way is to be robust: able to absorb a known setback. Another is to be repairable: easily modifiable in the face of unknown setbacks. Our approaches to threats, such as war or climate change, tend to be robust. What would a theory of repairability look like?

10. *The operating system of life*

Networks of gene regulation govern morphogenesis and determine cell identity. The concision of viruses suggests that this genetic software uses subroutines, like digital software. What are the laws governing genetic information processing? Can they shed light on the operating system of life, setting the stage for a biological analogue of the silicon revolution?

11. *The mathematical universe*

Wigner noted the unreasonable effectiveness of mathematics in physics. Today, we are seeing the reverse: attempts to advance physics, such as string theory, are driving mathematics. Is there a convergence between these two disciplines, and should this inform how much we fund and advance mathematics? Can Tegmark's mathematical universe be made rigorous?

12. *Describing network structure*

Network science, which extracts meaning from real-world networks, is popular but unsophisticated. To realize its potential, it must build on more rigorous concepts from graph theory and beyond. Can we formalise notions of network geometry and topology that are compatible with their continuum analogues, and a thermodynamics to describe deviations away from them?

13. *Theory of free will*

The existence of free will has no grounding in the laws of physics that are known. Some attempts have been made to link it to quantum phenomena, but a theory of free will remains elusive. Is it a phantom, a consequence of life, or a more general attribute of the present moment? What new physics is required to understand this seemingly vital concept?

14. *Collective creativity*

Collective creativity started 300 years ago when the scientific paper sped up research by enabling fast marginal advances over slow major ones. Now, anonymous collaboration platforms such as Wikipedia suggest we can speed up much more. But why and when does collective creativity work? Can platforms like Gowers' Polymath transform the process of discovery?

15. *Programmable matter*

We can cause surfaces and volumes to change shape on command using polymers that respond to temperature and current. What are the scope and limits of such programmable matter? Can we use differential geometry, recent advances in algorithmic origami and other mathematical tools to provide a language for reverse engineering useful shapes and mechanisms?

16. *Foundation of QFT*

Can quantum field theory, which describes all elementary particles and interactions, be made rigorous? An open problem is to prove that for any compact gauge group, a Yang-Mills theory exists in four dimensions and predicts a lightest particle with positive mass. This will likely require new kinds of mathematics and offer a new perspective on physics.

17. *Mathematical dualities*

Dualities play a key role in how we form insights in physics and mathematics. Examples include the Geometric Langlands correspondence, dualities across quantum field and string theories, and the ADE classification. Are dualities an artefact of how we decipher new theories, or do they have a more fundamental cause? Can we systematise them to discover more?

18. *Engineerable AI*

Evolution and innovation both make use of interoperable functional modules to increase the odds of successful tinkering. But deep learning algorithms, by contrast, are globally wired. This makes them difficult to build up in a hierarchical way, as well as hard for humans to understand. Can we formulate a framework for AI that is engineerable?

19. *Theory of simplicity*

In an increasingly complex world, we seek simplicity in how we organise technology and society. But we don't know how to describe simplicity, much less construct it. Physicists have developed various models of complexity, but what would a theory of simplicity look like? Is it connected to being able to readily reconfigure to new environments?

20. *AI-assisted conjectures*

Good conjectures can inspire new branches of mathematics. They come from spotting patterns and applying instinct. Because mathematics is exact and there are no equivalence coincidences, automated pattern detection is immune from the bias normally found in high dimensional search. Can machines help identify candidate conjectures and speed up theoretical research?

21. *Mathematics of causality*

Causality is fundamental to how we make predictions and structure society. Yet our mathematics for describing it is poor. Can a more sophisticated theory of causality help unlock challenges such as intelligent AI, the operating system of life and even how we build physical theories? How can we move from a microscopic to a macroscopic notion of causality?

22. *Emergence of virtue*

The fully-informed rational-agent basis of economics is inadequate for describing real-world behaviour, especially virtuous activity. Can insights from the microscopic view of behavioural science and the macroscopic view of thermodynamics form the basis of a cooperative game theory that accounts for the emergence of virtue in individuals and organisations?

23. *Theory of immortality*

Ageing is ascribed to the accumulation of errors—an inevitable consequence of the increase of disorder. But mounting experimental evidence suggests that ageing is not a fact of life. Is it a thermodynamic necessity, or is it instead favoured by natural selection? Can we mathematically describe the pros and cons of ageing? Is it possible to slow or even stop it?

Lecturas recomendadas

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